Deterministic vs. Stochastic Performance Assessment of Iterative Learning Control for Batch Processes

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The performance assessment of linear time-invariant batch processes when iterative learning control (ILC) is implemented has been discussed. Previous literatures show that conventional performance assessment cannot be directly applied to batch processes due to the nature of batch operations. Chen and Kong have suggested a new method to assess the control performance of batch processes using optimal ILC as the benchmark. In their work, ILC controllers are assumed to affect either stochastic or deterministic performance but without considering their interaction. This work elaborates the controllers effects on both stochastic and deterministic control performance of batch processes. It is shown that the optimal solution based on the minimum variance control law has a trade-off between deterministic and stochastic performance, which can be shown by a trade-off curve. Furthermore, a method is proposed to estimate this curve from routine operating data, against which the performance of ILC controllers can be assessed. Simulation studies are conducted to verify the proposed method. © 2012 American Institute of Chemical Engineers *AIChE J,* 59: 457–464, 2013
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Introduction

Because of increasing industrial applications of batch processes, there is a growing interest in investigating different control and monitoring methods[1] to improve control performance in terms of set-point tracking and disturbance rejection. Batch processes inherit large transient phases covering a wide range of operating envelopes. This implies that both the tracking and disturbance rejection problems must be addressed in any control design of batch processes.

To achieve the mentioned objectives, iterative learning control (ILC) has been widely attempted.[2–5] This method, first introduced by Arimoto et al.[6] for robot systems, considers the use of information from the previous batch to control the current one. At each batch, the input and error signals are kept for the use of the next batch. Design of ILC needs reliable models,[7–10] but in practice models have several kinds of uncertainties including parameter uncertainties or across-batch stochastic uncertainty, which have been discussed in literature.[11–15] To improve the performance of batch process control, combination of iterative learning algorithm with the model predictive control strategy has been proposed.[16,17] For certain aspects of ILC such as design and convergence, readers are referred to Verwoerd[18] and Moore.[19]

Since the global competition in chemical industry is increasing rapidly, performance assessment of control loops is receiving more attention as a means to ensure high efficiency of process operations. To assess the performance of a control loop, a benchmark is required as a reference. Minimum variance benchmark has gained popularity after it was first proposed by Harris,[20] based on the minimum variance control introduced by Astrom.[21] Astrom developed linear time-invariant minimum variance control, which was followed by remarkable progress in the fields of predictive control[22–24] and adaptive control.[25–27] Harris[20] suggested the use of minimum variance control as the benchmark. It was shown that there exists a feedback controller independent term in the process output that represents the minimum variance output and can be estimated from routine operating data. Since then, great progress in the area of performance assessment has been achieved.[28–37] The reason behind the popularity of this method is its simplicity and non-intrusiveness. It provides the best possible control, with respect to the output error variance, without relying on the process model or complicated computation process. This benchmark has also been extended to time-varying processes.[38–40] Theoretical and practical aspects of this subject have been well-covered by Huang and Shah.[41]

Performance assessment of continuous processes deals with disturbance rejection problem since the process often operates at a constant set-point. But for batch processes set-point tracking performance must also be taken into account. Qin[42] has categorized two different performance assessment problems. Chen and Kong[43] proposed a method to assess the performance of an ILC algorithm, that consists of two levels of controllers, by deriving the optimal control law for each controller. However, this method assumes that each of the two-level ILC controllers affects either the stochastic or deterministic control performance but without considering interaction.

This article elaborates the ILC controllers effects on both the stochastic and deterministic performance of batch

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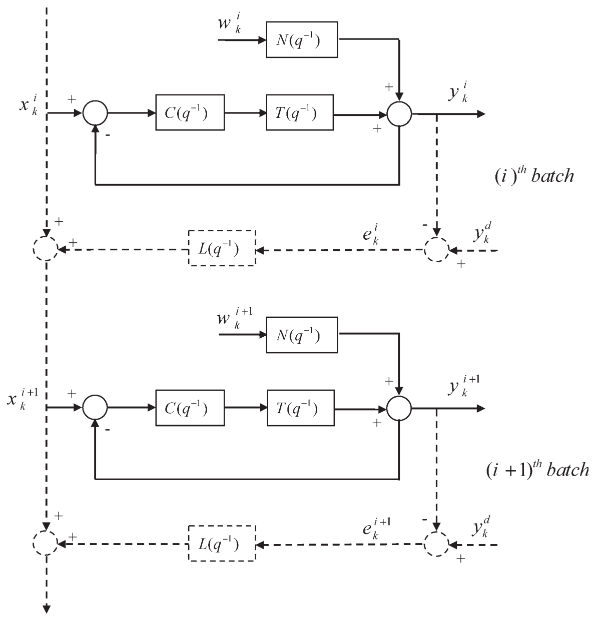

Figure 1. A recommended ILC structure.

control. For stochastic performance, the goal is to reject the disturbance, while for deterministic performance faster convergence is the goal. It is shown that there is a trade-off between convergence and disturbance rejection. A method is proposed to estimate the trade-off curve, which is then used as the benchmark to assess the performance of ILC controllers.

The remainder of this article is structured as follows: ILC set-up and a detailed explanation of its algorithm is shown in the Preliminaries. The optimal design of ILC controllers and estimation of the trade-off curve are discussed in the Optimal ILC Design. Performance assessment of the ILC controllers is addressed in the performance assessment. Simulation studies are conducted in simulation studies, followed by concluding remarks in the conclusions.

Preliminaries

Figure 1 illustrates an ILC algorithm that is applied to a process which operates over different batches with a constant batch period. Note that i, k, and q^{-1} represent batch index, time index, and time back-shift operator, respectively. As shown in Figure 1, the ILC algorithm consists of two loops. The inner loop, drawn with solid lines, operates similarly to a conventional control loop and includes the inner controller (C), plant model (T), and disturbance model (N). At each batch, the reference (x_k^i) is set by the outer loop, drawn with dashed lines. Disturbance is considered as another input to the process, which can be modeled by a colored noise derived from a white noise (w_k^i). After a batch is over, the output trajectory is compared with the set-point trajectory (y_k^d) and the error (e_k^i) is computed in the outer loop. The outer controller (L) filters the error trajectory that will then be added to the current reference to set the new reference for the next batch.

The outer loop sets the reference for each batch to reduce the error in the next batch. Under a good control policy, the process can track the set-point after a few batches. This is called convergence. For instance, in Figure 2, the third batch tracks the desired output, and its output error is caused only by stochastic disturbance. Hence, the designed control converges after three batches.

Equation 1 gives the linear relation between the inputs and output in a discrete-time transfer function format.

$$y_k^i = [1+TC]^{-1}TCx_k^i + [1+TC]^{-1}Nw_k^i \tag{1}$$

Note that for the sake of simplicity, the back-shift operator has been dropped. To cover non-linear dynamics of a batch process, nonlinear models or multilocal linear models for batch processes[44,45] may be considered along with the proposed performance assessment method. This article, will however, focus on linear models. To some extent, the inclusion of disturbances along with application of ILC strategy will handle some of the nonlinearity or time varying characteristic of a batch process.

The outer loop holds an update rule, as follows

$$x_k^i = x_k^{i-1} + Le_k^{i-1} \tag{2}$$

Let us define z^{-1} as the batch back-shift operator, as shown below

$$z^{-1}x_k^i = x_k^{i-1} \tag{3}$$

Hence Eq. 2 can be rearranged to give

$$x_k^i = (z-1)^{-1}Le_k^i \tag{4}$$

Optimal ILC Design

This section elaborates on the optimal design procedure for the ILC controllers based on the minimum variance control law. Error terms are derived and effects from both controllers on the error terms are discussed. To achieve desired output with respect to set-point tracking and disturbance rejection, control algorithms are developed.

General derivations

From the Diophantine identity,[20] we can write the closed-loop disturbance model in an impulse response form

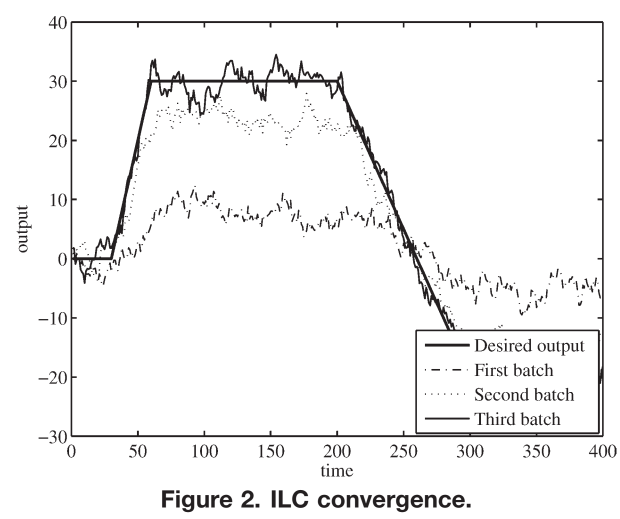

Figure 2. ILC convergence.

$$[1+TC]^{-1}N = F + Rq^{-d} \tag{5}$$

where F includes the first d terms of the impulse response and R includes the remainder. Substituting the closed-loop disturbance model with the Diophantine identity, Eq. 1 can be rewritten as

$$y_k^i = [1+TC]^{-1}TCx_k^i + [F + Rq^{-d}]w_k^i \tag{6}$$

The reference signal is set according to the update rule, presented in Eq. 4. Substituting the update rule into Eq. 6 yields

$$y_k^i = [1+TC]^{-1}TC(z-1)^{-1}Le_k^i + (F + Rq^{-d})w_k^i \tag{7}$$

Equation 7 can be rearranged to give

$$(z-1)y_k^i = [1+TC]^{-1}TCLe_k^i + (z-1)(F + Rq^{-d})w_k^i \tag{8}$$

For the sake of simplicity, let us define a new variable

$$G \triangleq [1+TC]^{-1}TCL \tag{9}$$

Substituting G into Eq. 8 yields

$$(z-1)y_k^i = Ge_k^i + (z-1)(F + Rq^{-d})w_k^i \tag{10}$$

which can be further simplified as

$$y_k^{i+1} - y_k^i = Ge_k^i + (F + Rq^{-d})(w_k^{i+1} - w_k^i) \tag{11}$$

By adding and subtracting the set-point to the left side of Eq. 11, the error can be written as a function of the previous batch error

$$y_k^{i+1} - y_k^i + y_k^d - y_k^d = Ge_k^i + (F + Rq^{-d})(w_k^{i+1} - w_k^i) \tag{12}$$

$$\rightarrow e_k^i - e_k^{i+1} = Ge_k^i + (F + Rq^{-d})(w_k^{i+1} - w_k^i) \tag{13}$$

$$\rightarrow e_k^{i+1} = (1-G)e_k^i + (F + Rq^{-d})(w_k^i - w_k^{i+1}) \tag{14}$$

Equation 14 gives the relation between the error sequences of different batches. To write the error as a function of the set-point and white noises, the error of the first batch is needed. For the first batch, it is common to use the set-point directly as the reference signal.[19] Let us rewrite Eq. 6 for the first batch

$$y_k^1 = [1+TC]^{-1}TCy_k^d + (F + Rq^{-d})w_k^1 \tag{15}$$

Subsequently, the error is found to be

$$\begin{aligned} e_k^1 = y_k^d - y_k^1 &= y_k^d - [1+TC]^{-1}TCy_k^d - [F + Rq^{-d}]w_k^1 \\ &= [1+TC]^{-1}y_k^d - (F + Rq^{-d})w_k^1 \end{aligned} \tag{16}$$

Having calculated the error expression of the first batch and according to Eq. 14, all the error terms can be derived, as shown below

$$e_k^2 = (1-G)[1+TC]^{-1}y_k^d + G(F + Rq^{-d})w_k^1 - (F + Rq^{-d})w_k^2 \tag{17}$$

$$\vdots$$

$$\begin{aligned} e_k^n = &(1-G)^{n-1}[1+TC]^{-1}y_k^d + (1-G)^{n-2}G(F + Rq^{-d})w_k^1 \\ &+ \cdots + (1-G)G(F + Rq^{-d})w_k^{n-2} + G(F + Rq^{-d})w_k^{n-1} \\ &- (F + Rq^{-d})w_k^n \end{aligned} \tag{18}$$

Since all the terms are derived from the set-point or white noises, the error of each batch can be split into deterministic and stochastic parts

$$e_k^n = e_k^{n,\mathrm{det}} + e_k^{n,\mathrm{sto}} \tag{19}$$

where

$$e_k^{n,\mathrm{det}} = (1-G)^{n-1}[1+TC]^{-1}y_k^d \tag{20}$$

$$\begin{aligned} e_k^{n,\mathrm{sto}} = &(1-G)^{n-2}G(F + Rq^{-d})w_k^1 \\ &+ \cdots + (1-G)G(F + Rq^{-d})w_k^{n-2} \\ &+ G(F + Rq^{-d})w_k^{n-1} - (F + Rq^{-d})w_k^n \end{aligned} \tag{21}$$

Consider two special cases:

1. $G = 0$

The outer controller is inactive, and the outer loop is, therefore, disconnected. Hence the reference signal remains constant across batches, and convergence cannot be achieved. From Eq. 18, the error can be computed as follows

$$e_k^n = [1+TC]^{-1}y_k^d - (F + Rq^{-d})w_k^n \tag{22}$$

2. $G = 1$

From Eq. 20, the deterministic error becomes zero after the operation of the outer loop, which implies that convergence will be achieved after one batch. From Eq. 18, the error is found to be

$$e_k^n = (F + Rq^{-d})(w_k^{n-1} - w_k^n), \quad n > 1 \tag{23}$$

Rate of convergence

Based on the definition, convergence of the ILC algorithm is achieved when the output error is caused only by stochastic disturbance, that is, the deterministic error becomes zero. Rate of convergence is defined as the number of batches required to achieve convergence, which can be used as a measure for the deterministic performance of the algorithm. The higher the rate of convergence, that is, the fewer batches required for convergence, the better the deterministic performance.

Let us define a new variable

$$\bar{y}_k^d \triangleq [1+TC]^{-1}y_k^d \tag{24}$$

which is the deterministic error of the first batch. Updating the deterministic error, Eq. 20, gives

$$e_k^{n,\mathrm{det}} = (1-G)^{n-1}\bar{y}_k^d \tag{25}$$

To achieve convergence, we need to equate Eq. 25 with zero

$$e_k^{n,\det} = (1-G)^{n-1}\bar{y}_k^d = 0 \tag{26}$$

Based on the expansion of Parseval's theorem,[46] we propose the following method to determine the convergence.

Expansion of Parseval's theorem: Let $u(t)$ and $y(t)$ be the input and output of a system with the following relation

$$y(t) = Hu(t) \tag{27}$$

where H is a discrete time transfer function. The following inequality holds between the two-norms of the input and output[46]

$$\|y\|_2 \leq \|H\|_\infty \|u\|_2 \tag{28}$$

Applying this theorem to Eq. 25 yields

$$\|e_k^{n,\det}\|_2 \leq \gamma_n \|\bar{y}_k^d\|_2 \tag{29}$$

where

$$\gamma_n = \|(1-G)^{n-1}\|_\infty \tag{30}$$

The algorithm is called converged after n batches, if the two-norm of the deterministic error of the nth batch decreases to less than $100\varepsilon\%$ of the two-norm of the first batch deterministic error, where $0 < \varepsilon \ll 1$. Therefore, the rate of convergence is equal to the smallest number of n that satisfies the following

$$\gamma_n \leq \varepsilon \tag{31}$$

Convergence Condition. In some cases, the ILC algorithm may never converge. When designing the algorithm, we need to consider the convergence property.

From the submultiplicative property of the ∞-norm,[46] we can expand Eq. 30

$$\gamma_n = \|(1-G)^{n-1}\|_\infty = \|(1-G)_\infty\|^{n-1} \tag{32}$$

If the following holds

$$\|(1-G)\|_\infty < 1 \tag{33}$$

Then Eq. 31 will certainly hold for a finite value of n. Otherwise, γ_n increases (or never decreases) after each batch and Eq. 31 will not hold. Therefore, Eq. 33 represents a sufficient condition of a convergent algorithm.

The inner controller

Recall the error expressions:

$$\begin{aligned} e_k^{n,\det} &= (1-G)^{n-1}[1+TC]^{-1}y_k^d \\ e_k^{n,\text{sto}} &= (1-G)^{n-2}G(F+Rq^{-d})w_k^1 + \cdots \\ &\quad + (1-G)G(F+Rq^{-d})w_k^{n-2} \\ &\quad + G(F+Rq^{-d})w_k^{n-1} - (F+Rq^{-d})w_k^n \end{aligned}$$

where G, F, and R are related in the following equations

$$\begin{aligned} G &= [1+TC]^{-1}TCL \\ F + Rq^{-d} &= [1+TC]^{-1}N \end{aligned}$$

It can be shown that F is the feedback controller independent term.[20] The outer controller affects only G, while the inner controller affects both G and R. Since the outer controller is capable of executing both causal and non-causal operators,[18,47] all the possible values of G can be reached by changing the outer controller. The inner controller can focus on changing R that is in the stochastic error expression. Therefore, the optimal inner controller can be derived by minimizing the stochastic error of the inner control loop. For this purpose, the regulatory control problem of the inner loop is studied. Assigning no reference signal to the inner loop, Eq. 1 can be written as

$$y_k^i = [1+TC]^{-1}Nw_k^i \tag{34}$$

Consider the following Diophantine identity

$$N = \acute{F} + \acute{R}q^{-d} \tag{35}$$

Substituting Eq. 35 into Eq. 34 yields

$$y_k^i = [1+TC]^{-1}(\acute{F} + \acute{R}q^{-d})w_k^i \tag{36}$$

$$= [1+TC]^{-1}(\acute{F} + TC\acute{F} + \acute{R}q^{-d} - TC\acute{F})w_k^i \tag{37}$$

$$= [\acute{F} + [1+TC]^{-1}(\acute{R}q^{-d} - TC\acute{F})]w_k^i \tag{38}$$

$$= \acute{F}w_k^i + [1+TC]^{-1}(\acute{R} - \tilde{T}C\acute{F})]q^{-d}w_k^i \tag{39}$$

where $\tilde{T}$ is the delay-free plant model. Since $\acute{F}$ includes the first d terms of the disturbance impulse response model, there are no correlations between the both terms of Eq. 39. Hence the first d terms of the disturbance impulse response model are independent of the inner controller. Therefore, we can conclude that $F = \acute{F}$ because F also represents the first d terms of the closed-loop disturbance model in impulse response form. Consequently, the variance of Eq. 39 can be written as

$$\text{var}(y_k^i) = \text{var}(Fw_k^i) + \text{var}([1+TC]^{-1}(\acute{R} - \tilde{T}CF)]q^{-d}w_k^i) \tag{40}$$

By optimizing Eq. 40 with respect to the inner controller, the following optimal inner controller is obtained

$$C^{\text{opt}} = \tilde{T}^{-1}\acute{R}F^{-1} \tag{41}$$

The outer controller

The outer controller affects both the deterministic and stochastic error terms. Among all the ILC algorithms with the same deterministic performance (the same rate of convergence), the one with the highest stochastic performance (the minimum variance of the stochastic error) is considered optimal. Therefore, a different optimal solution exists at each convergence rate.

To find the optimal solutions, the variance of the stochastic error needs to be minimized with respect to the outer

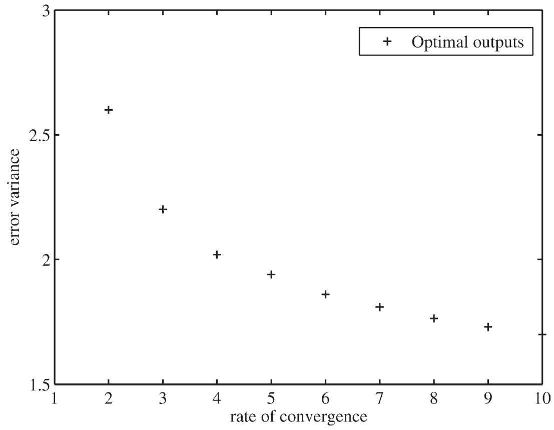

Figure 3. An example of the trade-off curve.

controller while the rate of convergence is set at the desired value. Note that the existing inner controller (not necessarily the minimum variance inner controller) is considered in the stochastic error term. The optimal solution for an ILC algorithm that converges after n batches meets two conditions:

1. The rate of convergence is n. From Eq. 31, the following holds

$$\gamma_n = \|(1-G)^{n-1}\|_\infty \le \varepsilon \tag{42}$$

2. The variance of the stochastic error, Eq. 21, is minimized. Originated from different white noises, all the terms of the stochastic error are uncorrelated. Therefore, the variance of the stochastic error is equal to

$$\begin{aligned}\mathrm{var}(e_k^{n,\mathrm{sto}}) &= \mathrm{var}((1-G)^{n-2}G(F+Rq^{-d})w_k^1) + \cdots \\ &+ \mathrm{var}((1-G)G(F+Rq^{-d})w_k^{n-2}) \\ &+ \mathrm{var}(G(F+Rq^{-d})w_k^{n-1}) + \mathrm{var}((F+Rq^{-d})w_k^n)\end{aligned} \tag{43}$$

To calculate the variance of each term, the following Lemma is used:

Lemma 1: *Given an expression $y(t) = Hw(t)$ where $w(t)$ is a white noise with variance of σ_w^2, the variance of $y(t)$ can be computed as follows*[48]

$$\mathrm{var}(y(t)) = \int_{-\pi}^{\pi} \phi_y(\omega)d\omega \tag{44}$$

where

$$\phi_y(\omega) = \frac{1}{2\pi}|H(e^{-j\omega})|^2\sigma_w^2 \tag{45}$$

This leads to

$$\mathrm{var}(y(t)) = \int_{-\pi}^{\pi}\frac{1}{2\pi}|H(e^{-j\omega})|^2 d\omega\sigma_w^2 = \|H\|_2^2\sigma_w^2 \tag{46}$$

Table 1. Optimal Controller and Minimum Variance Values for Case 1

Rate of Conv	2	3	4	5	6	7	8	9
b_0	0.789	0.54	0.404	0.322	0.267	0.228	0.201	0.192
b_1	0.039	0.058	0.046	0.037	0.028	0.033	0.031	0.029
f_1	0.143	0.253	0.281	0.293	0.293	0.340	0.362	0.376
Min. var.	2.404	2.001	1.839	1.754	1.702	1.667	1.641	1.622

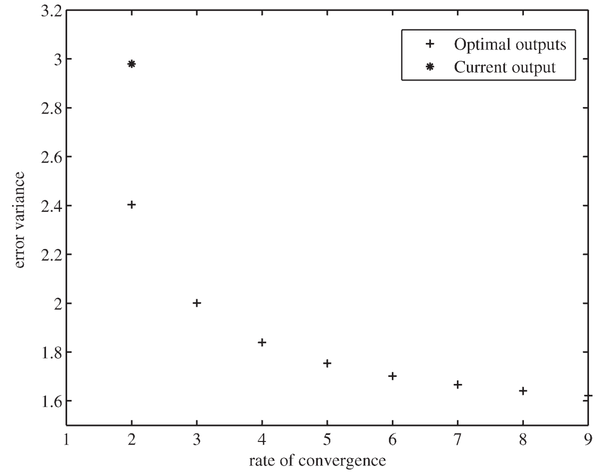

Figure 4. Trade-off curve for Case 1.

According to Lemma 1, the variance of the stochastic error is found to be

$$\begin{aligned}\mathrm{var}(e_k^{n,\mathrm{sto}}) = (&\|(1-G)^{n-2}G(F+Rq^{-d})\|_2^2 + \cdots \\ &+ \|(1-G)G(F+Rq^{-d})\|_2^2 \\ &+ \|G(F+Rq^{-d})\|_2^2 + \|(F+Rq^{-d})\|_2^2)\sigma_w^2\end{aligned} \tag{47}$$

Hence the following objective function is defined to minimize the variance of the stochastic error

$$J(G) = \min_G[\mathrm{var}(e_k^{n,\mathrm{sto}})] \tag{48}$$

$$\begin{aligned}= \min_G[&\|(1-G)^{n-2}G(F+Rq^{-d})\|_2^2 + \cdots \\ &+ \|(1-G)G(F+Rq^{-d})\|_2^2 \\ &+ \|G(F+Rq^{-d})\|_2^2 + \|(F+Rq^{-d})\|_2^2]\end{aligned} \tag{49}$$

Optimization Algorithm. To find the optimal solutions, we combine the two conditions into one objective function, as shown below

$$G^{\mathrm{opt}} = arg\min_G[\mathrm{var}(e_k^{n,\mathrm{sto}}) + \lambda|\gamma_n - \varepsilon|] \tag{50}$$

where λ is a weighting variable. As an example, consider a first-order structure for G^{opt}

$$G^{\mathrm{opt}} = \frac{b_0 + b_1q^{-1}}{1+f_1q^{-1}} \tag{51}$$

Optimization is performed with respect to three parameters of G^{opt}. For this example, the procedure is as follows:

1. Assign an integer to n.
2. A small initial value is set for λ.
3. A numerical method, like Newton-Raphson, is used to do the minimization.
4. If $\gamma_n \le \varepsilon$ (e.g., $\varepsilon = 0.05$) is satisfied, the solution is found. If not, continue to step 5.
5. A larger weighting variable is chosen, and step 2 is repeated.

Following the algorithm for different values of n, all the optimal solutions for the considered G^{opt} can be found. Plotting the stochastic error variance of all the solutions vs. the rate of convergence, we obtain a trade-off curve that

Table 2. Optimal Controller and Minimum Variance Values for Case 2

Rate of Conv	2	3	4	5	6	7	8	9
b_0	0.789	0.540	0.405	0.322	0.267	0.228	0.199	0.190
b_1	0.046	0.061	0.048	0.039	0.029	0.034	0.034	0.027
F_1	0.151	0.257	0.287	0.299	0.295	0.342	0.370	0.368
Min. var.	2.408	2.005	1.843	1.758	1.705	1.670	1.645	1.620

illustrates the trade-off between the deterministic and stochastic performance of the ILC algorithm.

For each optimal solution, the optimal outer controller can be computed from the inverse of the definition of G, as shown below

$$L^{\text{opt}} = (TC)^{-1}[1 + TC]G^{\text{opt}} \tag{52}$$

Performance Assessment

After collecting routine operating data from ILC controlled batch process, estimation of the process and disturbance models can be conducted. Let us rewrite the closed loop model of the inner loop, originally presented in Eq. 1

$$y_k^i = (1 - S)x_k^i + SNw_k^i \tag{53}$$

where

$$S = [1 + TC]^{-1} \tag{54}$$

Performing identification using output data y_k^i and the batch set-point reference data x_k^i, S and N are estimated. Since the existing inner controller is available, the plant model can also be estimated from S. F, R, and G are accordingly determined based on definitions, presented in Eq. 5 and Eq. 9.

The inner controller

As mentioned, the optimal inner controller minimizes the stochastic error of the inner loop. From Eq. 40, the minimum variance solution for the inner loop can be calculated from the first d terms of the disturbance impulse response model. For any existing inner controllers, substitute Eq. 5 into Eq. 34

$$y_k^i = (F + Rq^{-d})w_k^i \tag{55}$$

To determine the performance of the existing inner controller, an index is defined that is equal to the variance of the minimum variance solution for the inner loop divided by the variance of Eq. 55. Hence according to Lemma 1, the performance index of the inner controller can be determined as

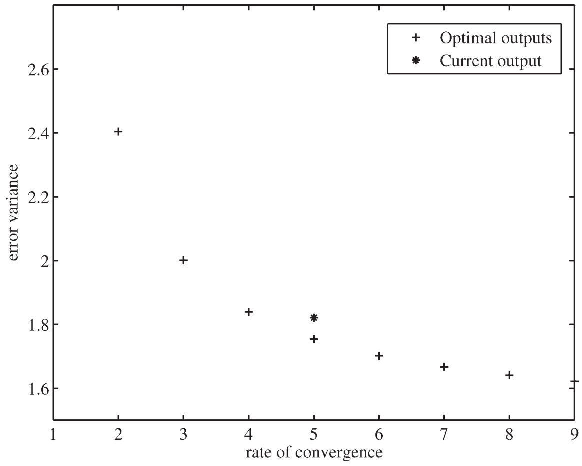

Figure 5. Trade-off curve for Case 2.

$$\eta_C = \frac{\|F\|_2^2}{\|F + Rq^{-d}\|_2^2} \tag{56}$$

which reflects the ratio between the optimal and existing performance. This index is the same as conventional performance index for feedback control of continuous processes.

The outer controller

Following the procedure for the optimal design of the outer controller for different convergence rates (n), a trade-off curve is obtained, as illustrated in Figure 3.

Having estimated F, R, and G, the rate of convergence and the stochastic error variance for the current ILC can be obtained from Eq. 31 and Eq. 47, respectively. If the current algorithm converges after n batches, the optimal solution with the rate of convergence equal to n is selected from the curve for performance assessment. The comparison of the variance of the current stochastic error with the variance of the optimal stochastic error for the same rate of convergence can offer a measure to determine the performance of the outer controller. Hence, we define an index equal to the variance of the stochastic error under the optimal outer control loop divided by the variance of the current stochastic error, as shown below

$$\eta_L = \frac{\|(1 - G^{\text{opt}})^{n-2}G^{\text{opt}}(F + Rq^{-d})\|_2^2 + \cdots + \|G^{\text{opt}}(F + Rq^{-d})\|_2^2 + \|(F + Rq^{-d})\|_2^2}{\|(1 - G)^{n-2}G(F + Rq^{-d})\|_2^2 + \cdots + \|G(F + Rq^{-d})\|_2^2 + \|(F + Rq^{-d})\|_2^2} \tag{57}$$

The numerator in Eq. 57 represents the solution under the optimal outer control loop. Note that since performance assessment of the outer controller is the objective, the existing inner controller (which is not necessarily optimal) is used for the calculation of the solution. Hence the difference in the error terms of the current and optimal solutions is caused only by the outer controller.

Simulation Studies

A batch process is given as below

$$T = \frac{0.9q^{-2} - 0.6q^{-3}}{1 - 0.7q^{-1}}$$
$$N = \frac{1 - 0.2q^{-1}}{1 - 0.9q^{-1}}$$

Table 3. Optimal Controller and Minimum Variance Values for Case 3

Rate of Conv	3	4	5	6	7	8	9	10
b_0	0.544	0.407	0.325	0.252	0.233	0.187	0.175	0.164
b_1	−0.007	−0.004	−0.003	−0.003	−0.002	−0.002	−0.002	−0.001
F_1	0.153	0.180	0.193	0.271	0.332	0.356	0.381	0.394
Min. var.	2.201	2.024	1.932	1.875	1.837	1.810	1.788	1.762

Three cases with different sets of ILC controllers are studied:

- *Case 1*

The optimal inner controller is implemented. The outer controller is chosen to achieve the fastest convergence, as shown below:

$$G = 1 \rightarrow L = (TC)^{-1}[1 + TC]$$

After collecting data, G, F, and R are estimated

$$\hat{F} = 1 + 0.7015q^{-1}$$
$$\hat{R} \simeq 0$$
$$\hat{G} = 1.004 - 0.007q^{-1}$$

To obtain the trade-off curve, the optimization procedure is applied. Table 1 shows the parameters of the optimal G and the minimum variance solutions (optimal stochastic error variance) under the optimal outer loop at different rates of convergence. Figure 4 also illustrates the trade-off curve and the current performance of the designed algorithm.

The current ILC algorithm converges after two batches with the error variance of 2.98. The minimum variance solution under the optimal outer loop at the same rate of convergence is 2.4041. Hence the performance index for the outer controller is

$$\eta_L = \frac{2.4041}{2.98} = 0.807$$

Based on Eq. 56, the performance index of the inner controller is 1, which is expected since the optimal inner controller is implemented.

- *Case 2*

The optimal inner controller and a relatively slower outer controller ($L = 0.5$) are implemented. Estimated G, F, and R are as follow

$$\hat{F} = 1 + 0.7033q^{-1}$$
$$\hat{R} \simeq 0$$
$$\hat{G} = 0.3173 + 0.053q^{-1} + 0.022q^{-2} + 0.016q^{-3} + 0.006q^{-4}$$

Table 2 shows the parameters of the optimal G and the minimum variance solutions under the optimal outer loop. The trade-off curve and the current performance of the designed algorithm are also presented in Figure 5.

The inner controller is the same as that in the previous case. Hence its performance index remains to be 1. This can be verified by calculating Eq. 56. The current ILC converges after 5 batches with the error variance of 1.821, while the minimum variance solution under the optimal outer loop for the same rate of convergence is 1.758. Hence the performance index of the outer controller is

$$\eta_L = \frac{1.758}{1.821} = 0.962$$

- *Case 3*

Two non-optimal inner and outer controllers are implemented, as shown below

$$C = \frac{0.5}{1 - 0.9q^{-1}}$$
$$L = 0.5$$

After obtaining data, G, F, and R are estimated

$$\hat{F} = 1 + 0.6983q^{-1}$$
$$\hat{R} \simeq 0.1836 - 0.1681q^{-1} - 0.2499q^{-2} - 0.1663q^{-3} - 0.043q^{-4} + 0.034q^{-5}$$
$$\hat{G} = 0.2241 + 0.205q^{-1} + 0.0913q^{-2} - 0.0097q^{-3} - 0.0578q^{-4} - 0.0532q^{-5} - 0.0243q^{-6}$$

Table 3 and Figure 6 show the parameters of the optimal G and the minimum variance solutions under the optimal outer loop, as well as the current performance of the designed algorithm.

The current performance indicates convergence after 10 batches with the error variance of 2.3129. The minimum variance solution under the optimal outer loop for the ILC algorithm with the same rate of convergence is 1.7627. Therefore, the performance index of the outer controller is

$$\eta_L = \frac{1.7627}{2.3129} = 0.762$$

Based on Eq. 56, the performance index of the inner controller can be determined as

$$\eta_C = \frac{1^2 + 0.6983^2}{1^2 + 0.6983^2 + 0.1836^2 + 0.1681^2 + \cdots} = 0.9$$

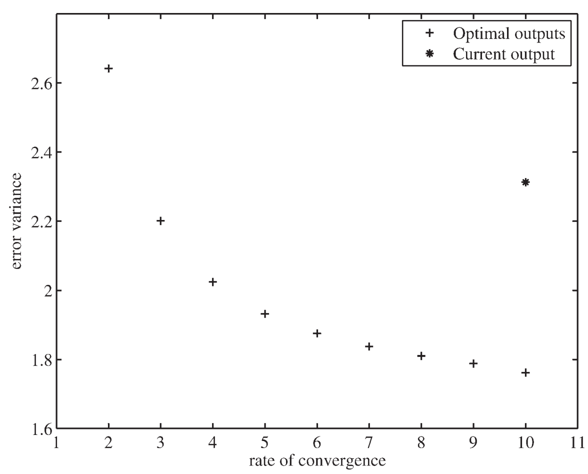

Figure 6. Trade-off curve for Case 3.

Conclusions

In this article, certain aspects of performance assessment for the ILC algorithm including identification, the extension of minimum variance benchmark, and performance assessment were discussed. First the ILC set-up and its algorithm were explained. Then the effects of both ILC controllers on the stochastic and deterministic performance were studied, followed by the optimal design procedure for each controller. The deterministic control performance of a process was determined by its rate of convergence, that is the number of batches required for convergence, while the stochastic control performance was determined by the stochastic error variance. Based on this consideration, it was shown that the solution of the inner loop under the optimal inner controller is equivalent to the conventional minimum variance solution. But for the outer controller, there is a trade-off between the deterministic and stochastic performance of control loops. Therefore, optimal solutions under the optimal outer controller were equivalent to the minimum variance solutions at given rates of convergence. All the optimal solutions were presented by a curve to demonstrate the mentioned trade-off. To compute the trade-off curve, error variance minimization was conducted to find the optimal solutions under the optimal outer controller for different rates of convergence. This article also elaborated the performance assessment of ILC controllers and verified the feasibility of the proposed methods through three simulation case studies.

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